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The Colonel Blotto Game

Transitivity and How to Win When You're an Underdog

- **The Colonel Blotto game is not as well known as the prisoner's dilemma, but offers some insights into strategic behavior nonetheless.**
- **The game shows why it is often impossible to legitimately crown a best team.**
- **Underdogs improve their chances of winning by changing the basis of competition.**
- **Successful investment strategies shift, which means even good long-term approaches fail from time to time.**

The X Factor

One of the most crucial players on a lacrosse team is the face-off specialist. To begin every quarter and after each goal, two opposing players go head-to-head in a face-off.[1] Since ball possession is vital to success in lacrosse, a dominant face-off specialist can make a huge difference in determining the game's outcome by keeping the ball out of the sticks of the opposing team's players.

Now imagine that you are part of the committee to select Team USA, the 23-man squad that will compete for the World Championship. Two players who are primarily face-off specialists were invited to the trials: Chris Eck, who played his college ball at Colgate University; and Alex Smith, a graduate of the University of Delaware. While Smith is the NCAA's all-time faceoff leader and widely considered one of the best face-off specialists in the world, Eck got the better of him in the trials. "I haven't gotten beat like this since high school," Smith lamented after Eck beat him in another draw. [2] With Eck winning fair and square, you would give him the roster spot, right?

Well, here's a complicating factor. The last World Championship Tournament was held in 2006, in which Team Canada upset Team USA in the finals, 15-10. Team Canada's most valuable player was Geoff Snider, a face-off specialist, who won a remarkable 19 of 28 face-offs in the final game. [3] Team USA needs a solution to Snider, who will be representing Canada again, since Team USA and Team Canada are the favorites to meet in the finals.

So while Eck may have Smith's number, the question is whether he can beat Snider. Since all three play in a professional league, we have the results of their interactions. Here they are:

- **Eck (A)** beat **Smith (B)** 59 percent of the time (34 of 58)
- **Smith (B)** beat **Snider (C)** 67 percent of the time (67 of 100)
- **Snider (C)** beat **Eck (A)** 52 percent of the time (38 of 73)

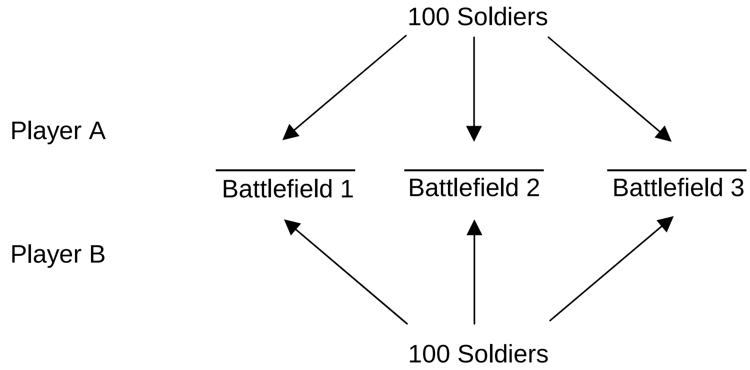
This is a case where there is a lack of transitivity. In a transitive condition, if A beats B and B beats C, then A beats C. But in this case A beats B, B beats C, but C beats A. If winning face-offs were transitive, there would be a "best" specialist. Not so here.

Team USA's coaching staff eventually went with Alex Smith for the final roster. On the one hand the decision makes sense given the likelihood that USA will have to get by Canada to win the tournament. On the other hand, it's hard to deny the spot on the roster to the guy who did better in camp.

Colonel Blotto to the Rescue

There is a model that is useful for thinking about this problem, as well as other challenges, called the Colonel Blotto game. [4] In the Colonel Blotto game, two players concurrently allocate resources across *n* battlefields. The player with greater resources in each battlefield wins that battle, and the player with the most overall wins is the victor. An extremely simple version of the game would have players A and B allocating 100 soldiers to three battlefields. Each player's goal is to employ a strategy that creates favorable mismatches versus his or her opponent. (See Exhibit 1.)

Exhibit 1: A Simple Colonel Blotto Game



A	30	30	40
B	33	33	34
Winner	**B**	**B**	**A**

Overall Winner: B

Source: LMCM analysis.

It is straightforward to see that Colonel Blotto is a zero sum game with multiple mixed strategy equilibria. In other words, so long as a player avoids very poor strategies (e.g., allocating all soldiers to one battlefield), this basic setup of the game resembles rock, paper, scissors. Not surprisingly, rock, paper, scissors is also a good way to describe how face-off specialists fare against one another.

Unlike the prisoner's dilemma, a well-known model in game theory, the Colonel Blotto game has had little impact on real-world decisions. Because the prisoner's dilemma has a preferred outcome (cooperation) and there is a good understanding of how cooperation emerges (repeated interactions), decision makers have been able to use the model in practical settings. [5] For example, the prisoner's dilemma nicely describes the emergence of the "live-and-let-live" system in trench warfare during World War I. [6] In numerous instances, both sides learned that there would be retaliation for any aggression. So when one side showed restraint, the other side learned to reciprocate by also showing restraint. This cooperation spared countless lives. The model has also been useful in international relations and business, among other areas. [7]

That the Colonel Blotto game is a mixed strategy game is part of the reason that it has had limited application to real-world interactions. Indeed, simple versions of the game provide little guidance for strategic interaction. However, the game does offer useful insights when you introduce more complex versions, including those with asymmetric resources or where the number of battlefields varies. In fact, asymmetric resources and multiple points of battle are common in zero-sum, strategic interactions. Simple illustrations include war, sports matches, and business.

The Colonel Blotto game is useful because by varying the game's two main parameters, giving one player more resources or changing the number of battlefields, you can gain insight into the likely winners of competitive encounters. It shows when underdogs have the best chance to win, why there is sometimes no "best" team, and how changes in the parameters influence those outcomes.

What a Modified Blotto Suggests

Let's look more closely at what happens when we alter the parameters. First, we can increase resource asymmetry by giving one player more points than the other, effectively making one side the favorite to win. It should come as no surprise that the stronger player wins more frequently. What's not as intuitive is how much of an advantage the additional points confer. In a three-battlefield game, a player with 25 percent more resources has a 60 percent expected payoff (the proportion of battles the player wins) and a player with twice the resources has a 78 percent expected payoff. So a decent dose of randomness exists even in contests with fairly asymmetric resources, though the resource-rich side has a decisive advantage.

But to get the whole picture of the payoffs we have to introduce the second parameter, the number of dimensions, or battlefields. The more dimensions the game has, the less certain the outcome (unless the players have identical resources) because the weaker player forces the stronger player to allocate resources more thinly, weakening the stronger player's relative advantage. For example, a weak player's expected payoff is nearly three times higher in a game with 15 dimensions than in a 9 dimension game. [8] In other words, by adding battlefields underdogs increase the number of interactions and improve the chances of an upset. Exhibit 2 summarizes how the changes in parameters influence outcomes.

Exhibit 2: High-Dimension Contests Increase the Uncertainty of Outcomes

Resource asymmetry		Dimensionality: Low	Dimensionality: High
	High	**Determinant outcomes** • Best player usually wins • Largely transitive	**Quasi-determinant outcomes** • Best player less dominant • Nontransitive
	Low	**Random outcomes** • Few suboptimal strategies • Nontransitive	**Random outcomes** • More suboptimal strategies • Nontransitive

Source: Michael J. Mauboussin, *Think Twice: Harnessing the Power of Counterintuition* (Boston, MA: Harvard Business Press, 2009), 94.

Blotto in the Real World

Here are some real-world cases where the Colonel Blotto game applies. The focus here is on two-player games where there are asymmetric resources and some strategic ability to change the number of battlefields. Many of the core lessons likely apply for multi-player interactions as well, including products competing on the store shelf or colleges seeking to increase their status in the rankings of the best schools.

One realm where the Colonel Blotto game is germane is asymmetric conflict—wars between strong and weak actors. There is a rich tradition of celebrating wins by the weak, including the biblical story of David and Goliath. It is notable that the younger and weaker David shunned a traditional battle using a helmet and sword and chose instead to fight unconventionally with stones and a slingshot. [9] He had to change the basis of competition to have a chance at victory.

In his book, *How the Weak Win Wars*, Ivan Arreguín-Toft, a political scientist at Boston University, analyzed roughly 200 asymmetric conflicts from 1800 to 2003. He coded a conflict as asymmetric if the stronger actor's resources (forces and population) exceeded those of the weaker actor by a factor of ten or more. His analysis reveals two basic findings. First, the stronger actor prevailed just 72 percent of the time. Since this only included conflicts where the asymmetry in resources was large, the success of the weaker players is noteworthy.

Second, over the past two centuries the weaker players have been winning at a higher and higher rate. For instance, strong actors prevailed in 88 percent of the conflicts from 1800 to 1849, but the rate dropped to very close to 50 percent from 1950 to 1999. Further, the weak actors saw their percentage of victories rise in each of the 50-year sub periods from 1800 to 1999.

After reviewing and dismissing a number of possible explanations for these findings, Arreguín-Toft suggests that an analysis of strategic interaction best explains the results. Specifically, when the strong and weak actors go toe-to-toe (effectively, a low *n*), the weak actor loses roughly 80 percent of the time because "there is nothing to mediate or deflect a strong player's power advantage." In contrast, when the weak actors choose to compete on a different strategic basis (effectively increasing the size of *n*), they lose less than 40 percent of the time "because the weak refuse to engage where the strong actor has a power advantage." [10] Weak actors have been winning more conflicts over the years because they see and imitate the successful strategies of other actors and have come to the realization that refusing to fight on the strong actor's terms improves their chances of victory. [11]

What the analysis also reveals, however, is that nearly 80 percent of the losers in asymmetric conflicts never switch strategies. Part of the reason players don't switch is that there is a cost: when personnel training and equipment are geared toward one strategy, it's often costly to shift to another. New strategies are also stymied by leaders or organizational traditions. This type of inertia appears to be a consequential impediment to organizations embracing the strategic actions implied by the Colonel Blotto game.

Another case where the Colonel Blotto game is illustrative is sports competition. Like war, two teams that often have asymmetric resources compete in what is generally a zero-sum game (some sports do allow ties). Resources are asymmetric if one side has a player or players who are quicker, stronger, fitter, or better-skilled than the competition. Battlefields can be thought of as discrete interactions within the game—for instance, the quality of serves versus the return of serves in tennis, or passing offense versus passing defense in American football.

An analysis of the statistics reveals that sports with a greater degree of interactions have a larger percentage of instances where weaker teams beat stronger teams. Past win-loss records capture asymmetry here, and the number of players approximate interactions. For instance, a study of over 43,000 games revealed that the underdog won approximately 45 percent of the time in the English Football Association—England's premier soccer league—but a lower 37 percent of the time in the U.S.'s National Basketball Association.[12] This empirical observation fits well with the Colonel Blotto game, which demonstrates that more battlefields improve the odds that the underdog will win.

A more concrete example comes from Division I college football. Texas Tech has adopted a strategy that has allowed it to win over 70 percent of its games in recent years despite playing a highly competitive schedule. The team's success is particularly remarkable since few of the players were highly recruited or considered "first-rate material" by the professional scouts. Based on personnel alone, the team was weaker than many of its opponents. [13]

Knowing that employing a traditional game plan would put his weaker team at a marked disadvantage, the coach offset the talent gap by introducing more complexity into the team's offense via a large number of formations. These formations change the geometry of the game, forcing opponents to change their defensive strategies. It also creates new matchups (i.e., increasing *n*, the number of battlefields) that the stronger teams have difficulty winning. For example, defensive linemen have to drop back to cover receivers. The team's coach explained that "defensive linemen really aren't much good at covering receivers. They aren't built to run around that much. And when they do, you have a bunch of people on the other team doing things they don't have much experience doing." This approach is considered unusual in the generally conservative game of college football.

These illustrations from asymmetric conflict in war and sports show that increasing the number of battlefields can have a meaningful impact on outcomes. That weaker players are generally well-served to increase the number of battlefields is a valuable observation that has real-world applications. Remarkably, though, weak players in many domains choose to play a conventional game rather than incur the cost (which can often be reputational as well as financial) of adding battlefields.

Why Investors Should Care

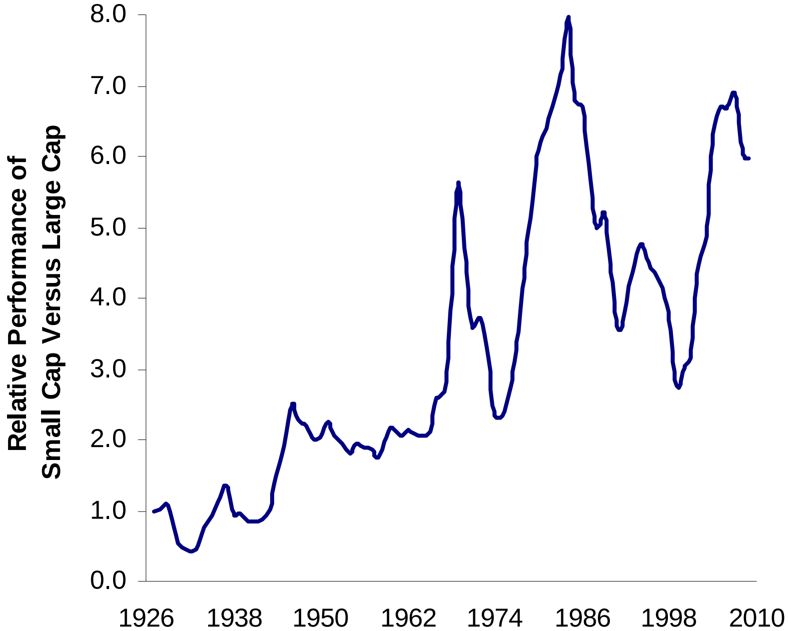
So what lessons can investors learn from studying the Colonel Blotto game? The first important one is how to compete when you are the underdog. Theory and practice show that when you have fewer resources than your foe in a head-to-head competition, you should compete in a non-traditional way in order to expand the number of battlefields. Since adding battlefields frequently conflicts with conventional wisdom, competitors do not chose the approach as often as they should. But the significance of using a non-traditional basis of competition is highly relevant in games and business.

Another lesson is that in many competitive situations, there is no "best" team, player, or strategy. [14] The winner of a particular matchup, game, or tournament depends as much on the allocation of strengths and weaknesses as on their magnitude. This is why the selection of Team USA's face-off specialist was so difficult: the best player at the trials may not have been the best player for the tournament. So while many people—and Westerners in particular—are keen to crown one competitor as king, the reality is that success relies heavily on the competitive circumstances.

Finally, Colonel Blotto's lessons apply directly to the investment world. Investors generally embrace a particular investment strategy—value, growth, small capitalization, etc.—because they believe that it will generate excess returns over time. But, as you would guess, different strategies work at different times in the market cycle. [15]

One illustration is the results of small capitalization versus large capitalization stocks. In 1981, Rolf Banz published an influential paper showing that small-cap stocks delivered higher risk-adjusted returns than large-cap stocks from 1926-1975. [16] Updated through 2009, Banz's findings hold true. However, there are long stretches during which large-cap stocks outperform small-cap stocks. [17] Exhibit 3 shows the results of small caps versus the results for large caps. Had you implemented Banz's findings as a strategy in the early 1980s, you would have suffered almost two decades of relatively poor returns.

Exhibit 3: Regime Changes between Large Cap and Small Cap Stocks



Source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html; LMCM analysis.

But had your experience of the 1980s and 1990s cemented your view that large caps were the way to go, the first decade of the twenty-first century would have proven you wrong.[18] While difficult to say, the odds now appear to favor large cap stocks again, based on a combination of relatively sluggish performance (a flat S&P 500 for nearly a decade) and attractive valuation.

Endnotes

[1] This is not true in all instances. For example, a team that is "man up" because the other team is serving a penalty automatically gets the ball back at the start of the next quarter provided they have possession at the end of the quarter.
[2] Matt DaSilva, "Great X-Pectations," *Lacrosse Magazine*, October 2009, 11.
[3] See http://www.pointstreak.com/prostats/boxscore.html?gameid=337469. A win percentage of 55 percent is considered very good, and over 60 percent is exceptional. In the final game, Snider won almost 70 percent of the draws. During the whole tournament, he won over 73 percent of his face-offs.
[4] For a formal treatment of the Colonel Blotto game, see Brian Roberson, "The Colonel Blotto Game," *Economic Theory*, Vol. 29, No. 1. September 2006, 1-24; for a more informal discussion, see Scott E. Page, *The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies* (Princeton, NJ: Princeton University Press, 2007), 112-114; and Jeffrey Kluger, *Simplexity: Why Simple Things Become Complex (*and *How Complex Things Can Be Made Simple)* (New York: Hyperion, 2008*)*, 183-185.
[5] Russell Golman and Scott E. Page, "General Blotto: Games of Allocative Strategic Mismatch," *Public Choice*, Vol. 138, No. 3-4, March 2009, 279-299.
[6] Robert Axelrod, *The Evolution of Cooperation* (New York: Basic Books, 1984).
[7] For international relations, see William Poundstone, *Prisoner's Dilemma* (New York: Doubleday, 1992); for business, see Avinash K. Dixit and Barry J. Nalebuff, *The Art of Strategy* (New York: W.W. Norton & Company, 2008).
[8] For this example I selected an Xa/Xb ratio of 0.13. Using Theorem 3 from Roberson (2006), the expected payoff is 2.5 percent when n equals 9. Using Theorem 2, the expected payoff is 6.7 percent when n equals 15.
[9] Robert Alter, *The David Story* (New York: W.W. Norton & Company, 1999).
[10] Ivan Arreguín-Toft, *How the Weak Win Wars* (Cambridge: Cambridge University Press, 2005). This book is mentioned in a popular article: Malcolm Gladwell, "How David Beats Goliath: When Underdogs Break the Rules," *The New Yorker*, May 11, 2009.
[11] Kenneth Waltz, *Theory of International Politics* (New York: McGraw Hill, 1979).

[12] Eli Ben-Naim, Federico Vazquez, and Sidney Redner, "Parity and Predictability of Competitions," *Journal of Quantitative Analysis in Sports,* Vol. 2, No. 4, 2006.
[13] Michael Lewis, "Coach Leach Goes Deep, Very Deep," *The New York Times*, December 4, 2005.
[14] Page, 114.
[15] Two points are worth noting here. First, some strategies may do better over very long periods of time. But the horizon over which a strategy proves successful and the horizon over which investors evaluate fund managers may be very different. So a fund manager may fail with a successful long-term strategy if capital providers are impatient. Second, the market itself gives clues about when strategies may shift. For example, the combination of excellent results for an asset class and a resulting valuation that is at the high end of historical bounds frequently suggests poor future returns.
[16] Rolf W. Banz, "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics*, Vol. 9, No. 1, March 1981, 3-18.
[17] See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
[18] Gerald P. Madden, Kenneth P. Nunn, Jr., and Alan Wiemann, "Mutual Fund Performance and Market Capitalization," *Financial Analysts Journal*, Vol. 42, No. 4, July-August 1986, 67-70 and Jeff D. Opdyke, "Small Caps Looms Large," *The Wall Street Journal*, May 1, 2010.